Advanced Placement Calculus AB

2021-2022

Summer Assignment

Greetings! Here we are at the end of the 2020-2021 school year. Soon you will be returning, having to tackle one of the most challenging and rewarding classes that you can take in high school. I have some advice. You can take it or leave it, but it is wisdom based on 12 years of seeing students struggle because of choices that they are making. Here it is: Get a lot of rest and get mentally prepared to be challenged. Think about your schedule next year and consider what you can handle and be excellent at. You will be better off if you can learn to be excellent in a few things rather than mediocre in many things. Students who take on too many tasks involving sports, jobs, school plays, and extra-curricular activities struggle in Calculus. They do not have enough time to excel. I am not saying to get rid of all activities, but Calculus will need your attention next year to the tune of 1-2 hours a night. So, get your schedule in order before you return to school and be prepared to work daily to stay on top of the work load.

By the time August rolls around, begin working on the Summer Assignment. It will help you review past material and become familiar with mathematical concepts that we will encounter in Calculus. The following steps will help guide you.

First, read through section 1 of this packet. It contains the material that you need to know and understand. It should look familiar and will be a valuable resource for you as you do your work in section 4 and throughout the upcoming year.

Second, go online (the Internet), and search for math sites that you think might be helpful for you this year. See if you can come up with 5-10 websites. I'll give you one: www.khanacademy.org can help you with your math skills (like what you need to do in section 4 of this packet.)

Third, I'd like you to perform the simple experiment in section 3. You might need a buddy to help you with this. Of course, practice social distancing. So perhaps you choose a family member to help you instead.

Finally, complete the algebra practice in section 4. Like most students, you may find that you are rusty with some of the problems that I have given you. If so, refer to the helpful resources on the internet that you have found. You may also work with other students in your class, but BE CAREFUL. Do not split the assignment up to make it less work. You need to be sure that YOU complete each problem yourself and that YOU fully understand how to solve each one. I will give you fair warning. Students who get A's in Calculus do every single assigned problem. There are no shortcuts to learning Calculus.

I will collect sections 3&4 on the first day of school. You must show the steps you took to solve each problem. Just an answer will earn no credit.

I look forward to working with you next year. I think that you will find my classes lighthearted and fun, yet difficult and challenging.

P.S. If you get stuck, send me an email (kevin.winter@sno.wednet.edu). If this seems overwhelming, don't panic and certainly don't quit on Calculus. Just get in touch with me and I'll gladly help you out. This goes for the entire year.

Best regards,

Mr. Winter

Section 1 (pg.1) - Review

Stuff You Must Know Cold from Algebra 1

Powers of 2

$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024$
$2^{11} = 2048$
$2^{12} = 4096$

Squares

$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = 196$
$15^2 = 225$
$16^2 = 256$
$17^2 = 289$
$18^2 = 324$
$19^2 = 361$
$20^2 = 400$
$21^2 = 441$
$22^2 = 484$
$23^2 = 529$
$24^2 = 576$
$25^2 = 625$

Cubes

$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$
$11^3 = 1331$
$12^3 = 1728$

Factorials

$0! = 1$
$1! = 1$
$2! = 2$
$3! = 6$
$4! = 24$
$5! = 120$

Inequality Meanings

$<$ *less than*

$\leq$ *less than or equal to*

$>$ *greater than*

$\geq$ *greater than or equal to*

Linear Equations

Parent Function:

$$y = x$$

Standard Form:

$$ax + by = c$$

Slope-Intercept Form:

$$y = mx + b$$

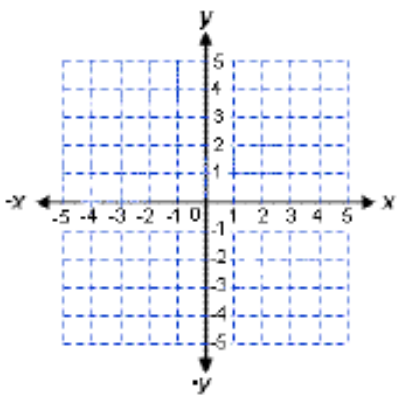
Point-Slope Form:

$$y - y_1 = m(x - x_1)$$

Slope:

$$\frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Graph: $y =$
(label 3 points)



Section 1 (pg.2) - Review

Stuff You Must Know Cold from Geometry

Quadratic Equations

Parent Function:

$$y = x^2$$

Standard Form (Vertex Form):

$$y = a(x-h)^2 + k$$

General Form:

$$y = ax^2 + bx + c$$

Vertex:

$$(h, k)$$

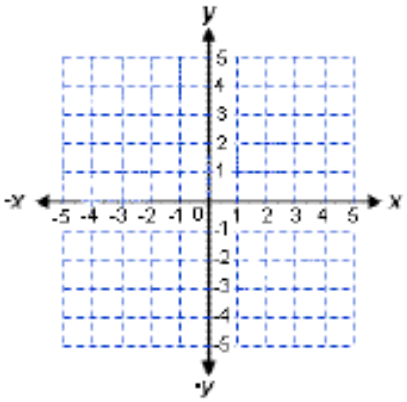
Axis of Symmetry:

$$x = \frac{-b}{2a}$$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graph: $y =$
(label 3 points)



Order of Operations

1. Brackets
2. Exponents
3. Multiply & Divide (left to right)
4. Add & Subtract (left to right)

Function Definitions

Domain: *The set of all possible input values (usually x)*

Range: *The set of all possible output values (usually y)*

Function: *A relation where each element in the domain (x) matches with exactly one element of the range (y)*

Direct Variation:

$$y = kx$$

Indirect Variation:

$$y = \frac{k}{x}$$

Roots:

The x-intercepts of a function, where the function (y) equals zero. Roots are also known as solutions, zeros, and x-intercepts.

Midpoint Formula

$$(m_1, m_2) =$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Use a, b, and c to describe the following properties:

Commutative Property

Addition:

$$a + b = b + a$$

Multiplication:

$$a \cdot b = b \cdot a$$

Associative Property

Addition:

$$a + (b + c) = (a + b) + c$$

Multiplication:

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

Distributive Property

$$a(b + c) = ab + ac$$

Absolute Value

$a \geq 0$

$|a| = a$

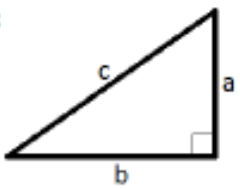
$|-a| = a$

Distance Formula

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Pythagorean Theorem

$$c^2 = a^2 + b^2$$



Section 1 (pg.3) - Review

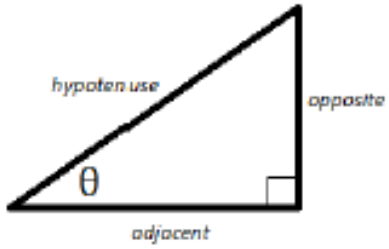
Stuff You Must Know Cold from Trigonometry

Trigonometric Ratios

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$



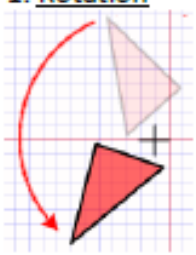
Triangle Congruence

1. Side-Side-Side *(SSS)*
2. Side-Angle-Side *(SAS)*
3. Angle-Side-Angle *(ASA)*
4. Angle-Angle-Side *(AAS)*
5. Hypotenuse-Leg *(HL)*

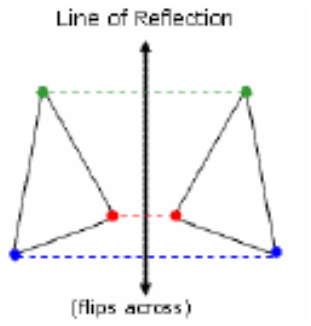
http://www.mathopenref.com/congruenttriangles.html

Transformations

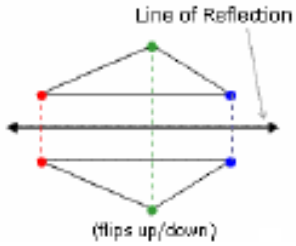
1. Rotation



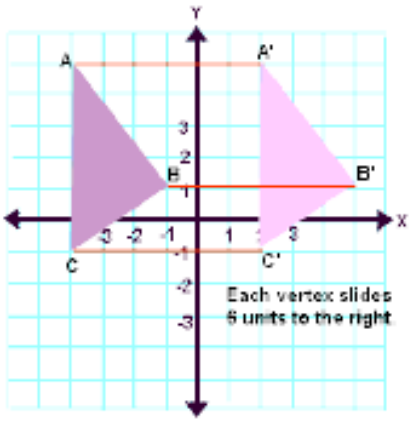
2. Horizontal Reflection



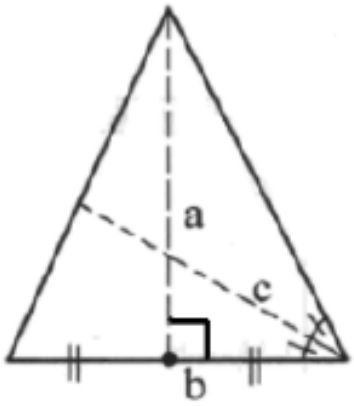
3. Vertical Reflection



4. Translation



Parts of a Triangle



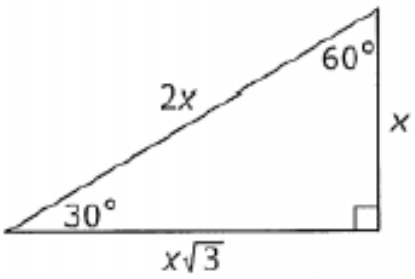
a: *altitude (height)*

b: *base (with midpoint)*

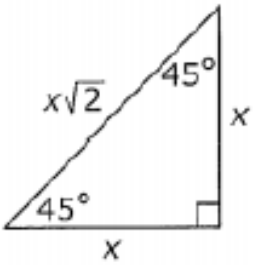
c: *angle bisector*

Special Right Triangles

30° – 60° – 90° triangle



45° – 45° – 90° triangle



Similarity

Ratio of Sides: $m:n$

Ratio of Perimeters: $m:n$

Ratio of Areas: $m^2:n^2$

Ratio of Volumes: $m^3:n^3$

Section 1 (pg.4) - Review

Other Stuff You Must Know Cold from Geometry

Perimeter Formulas

Square: $P = 4s$

Rectangle: $P = 2l + 2w$

Circumference: $C = \pi r^2$

Roots to Know

$\sqrt{2} \approx 1.41$

$\sqrt{3} \approx 1.73$

Area Formulas

Square: $A = s^2$

Rectangle: $A = l \cdot w$

Parallelogram: $A = b \cdot h$

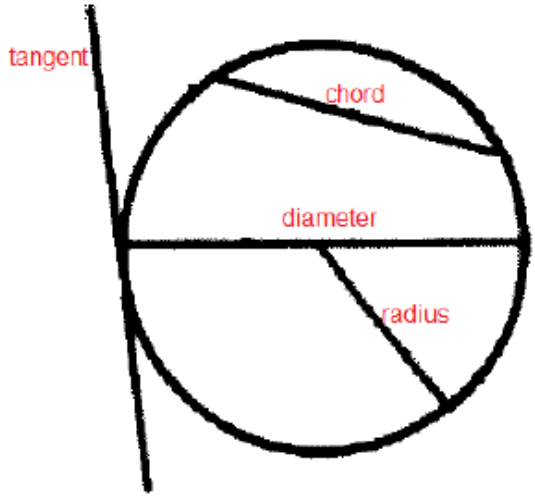
Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Circle: $A = \pi r^2$

Triangle: $A = \frac{1}{2}b \cdot h$

Regular Polygon: $A = \frac{1}{2}a \cdot p$

Parts of a Circle



Surface Area Formulas

Cube: $S = 6s^2$

Sphere: $S = 4\pi r^2$

Cylinder: $S = 2B + Ch$

Volume Formulas

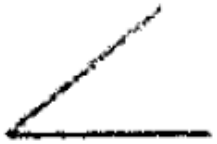
Cube: $V = s^3$

Prism/Cylinder: $V = B \cdot h$

Pyramid/Cone: $V = \frac{1}{3}B \cdot h$

Sphere: $V = \frac{4}{3}\pi r^3$

Types of Angles:



is acute

is obtuse

Complementary angles add up equal 90° .

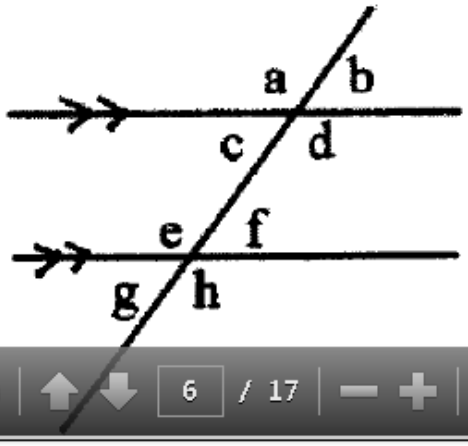
Supplementary angles add up equal 180° .

Sum of Interior Angles

Triangle = 180°

Quadrilateral = 360°

Regular Polygon = 180° (n-2)



Angle a is congruent to:

Angles d, e, and h

Angle a is supplementary to:

Angles b, c, f, and g

Section 1 (pg.5) - Review

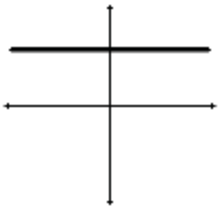
www.khanacademy.org/math/algebra/algebra-functions

Toolkit of Functions

Students should know the basic shape of these functions and be able to graph their transformations without the assistance of a calculator.

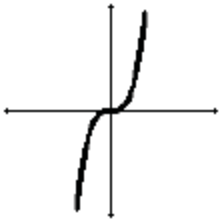
Constant

$f(x) = a$



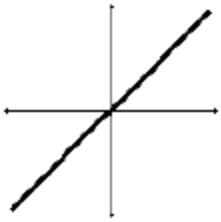
Cubic

$f(x) = x^3$



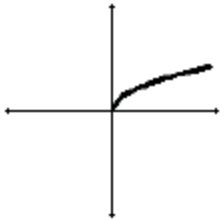
Identity

$f(x) = x$



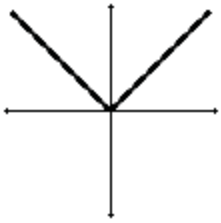
Square Root

$f(x) = \sqrt{x}$



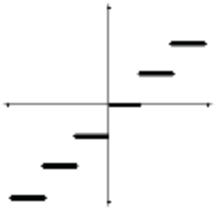
Absolute Value

$f(x) = |x|$



Greatest Integer

$f(x) = [x]$



Reciprocal

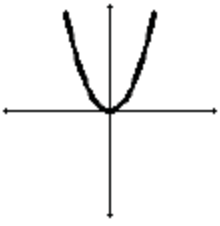
$f(x) = \frac{1}{x}$

Exponential

$f(x) = a^x$

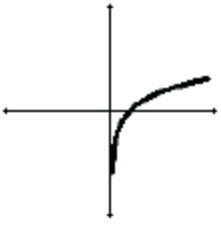
Quadratic

$f(x) = x^2$



Logarithmic

$f(x) = \ln x$

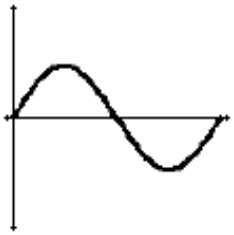


Section 1 (pg.6) - Review

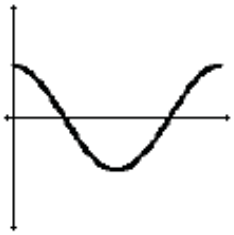
http://www.khanacademy.org/math/trigonometry

Trig Functions

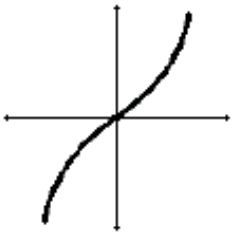
$f(x) = \sin x$



$f(x) = \cos x$



$f(x) = \tan x$



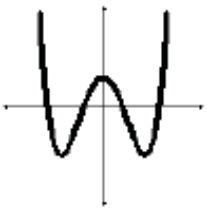
Polynomial Functions:

A function P is called a polynomial if $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$
Where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots a_n$ are constants.

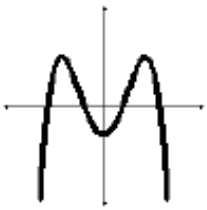
Even degree

Leading coefficient sign

Positive



Negative



Odd degree

Leading coefficient sign

Positive

Negative

- Number of roots equals the degree of the polynomial.
- Number of x intercepts is less than or equal to the degree.
- Number of "turns" is less than or equal to (degree – 1).

Section 1 (pg.7) - Review

Trigonometric Identities

** Memorize the marked families of Identities

**** Reciprocal Identities**

$$\sin X = \frac{1}{\csc X}$$
$$\cos X = \frac{1}{\sec X}$$
$$\tan X = \frac{1}{\cot X}$$
$$\csc X = \frac{1}{\sin X}$$
$$\sec X = \frac{1}{\cos X}$$
$$\cot X = \frac{1}{\tan X}$$

**** Quotient Identities**

$$\tan X = \frac{\sin X}{\cos X}$$
$$\cot X = \frac{\cos X}{\sin X}$$

**** Pythagorean Identities**

$$\sin^2 X + \cos^2 X = 1$$
$$1 + \tan^2 X = \sec^2 X$$
$$1 + \cot^2 X = \csc^2 X$$

**** Even Identities**

$$\cos(X) = \cos(-X)$$
$$\sec(X) = \sec(-X)$$

**** Odd Identities**

$$\sin(-X) = -\sin(X)$$
$$\csc(-X) = -\csc(X)$$
$$\tan(-X) = -\tan(X)$$
$$\cot(-X) = -\cot(X)$$

Co-Function Identities

$$\sin\left(\frac{\pi}{2} - X\right) = \cos(X)$$
$$\cos\left(\frac{\pi}{2} - X\right) = \sin(X)$$
$$\tan\left(\frac{\pi}{2} - X\right) = \cot(X)$$
$$\cot\left(\frac{\pi}{2} - X\right) = \tan(X)$$
$$\sec\left(\frac{\pi}{2} - X\right) = \csc(X)$$
$$\csc\left(\frac{\pi}{2} - X\right) = \sec(X)$$

Power Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$
$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$
$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$
$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$
$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$
$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Double Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$
$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$
$$\cos 2u = \cos^2 u - \sin^2 u$$
$$\cos 2u = 2 \cos^2 u - 1$$
$$\cos 2u = 1 - 2 \sin^2 u$$

Half Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}}$$
$$\cos\frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$
$$\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

The signs of $\sin\frac{u}{2}$ and $\cos\frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

Product to Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$
$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

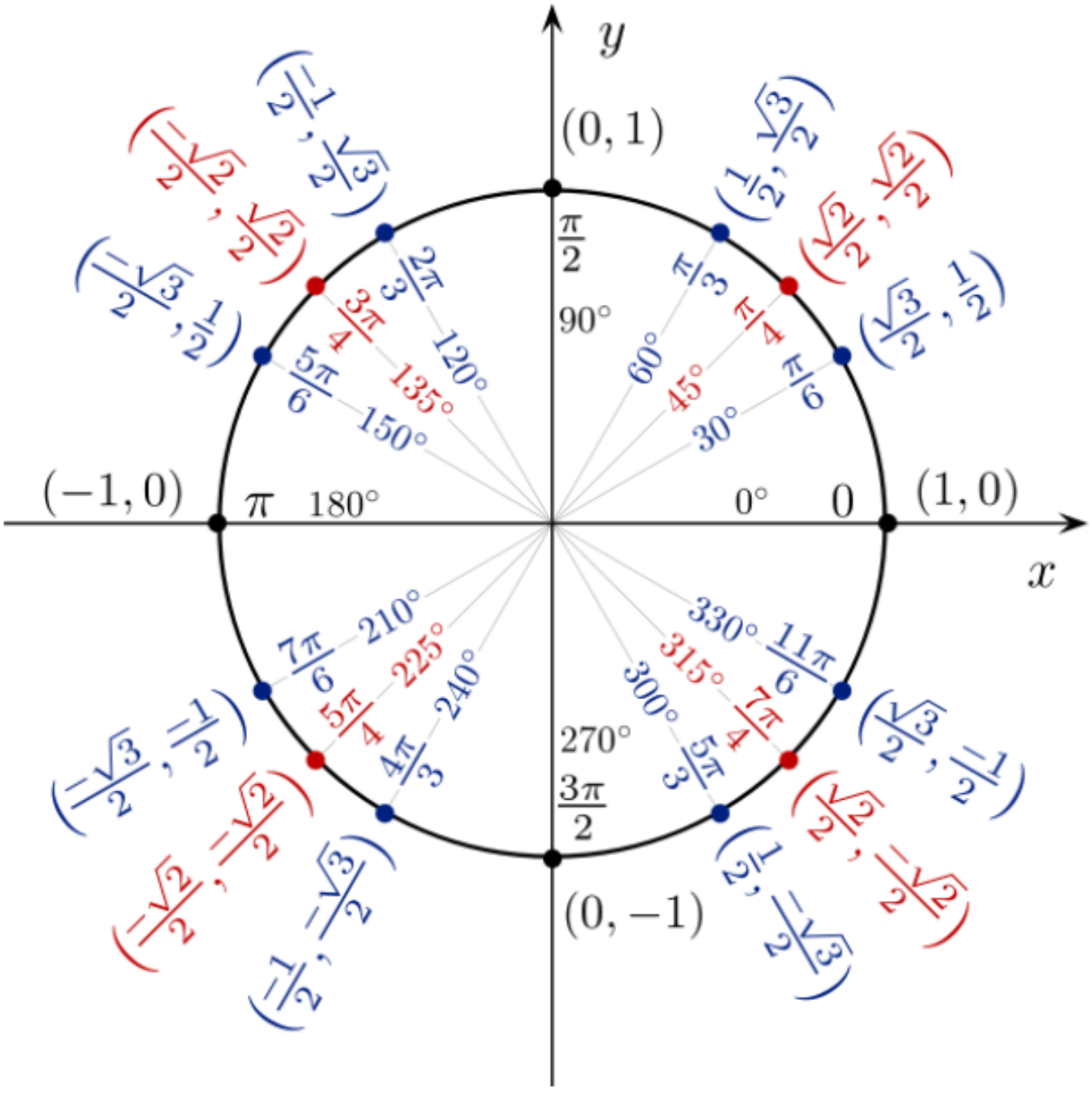
Sum to Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$
$$\sin u - \sin v = 2 \cos\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$
$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$
$$\cos u - \cos v = -2 \sin\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$

Section 1 (pg.8) - Review

Unit Circle

You must be able to use the unit circle... from memory!



Section 2 – Helpful Websites

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

Section 3 –Experiment

For this experiment, you will need a ball (a tennis ball will work great, a bowling ball not so much), and a timer. You may get help from a friend or parent if you'd like. Use these definitions to help you answer the following questions.

I) **Position** includes the distance (length) and direction (this sign +/-) of an object from a starting reference point. (Your hand will be the starting reference point and will have a position of 0 ft.) When the ball is above your hand, its direction is positive. When the ball is below your hand, its direction is negative.

II) Velocity is the change in position over the change in time $\left(V = \frac{\Delta position}{\Delta time}\right)$. Velocity includes the speed (ft/s) and direction of an object. If the ball is moving upward, its velocity if positive. If the ball is moving downward its velocity is negative. Notice that speed and velocity are not the same. Speed is always positive, where velocity can be positive or negative.

III) Acceleration is the change in velocity over the change in time $\left(a = \frac{\Delta velocity}{\Delta time}\right)$. The sign of acceleration is a bit tricky because we usually think of it as speeding up or slowing down. Here is some helpful guidelines to use. If the speed is increasing in a positive direction, then acceleration is positive. If the speed is decreasing in a positive direction, then acceleration is negative. If the speed is increasing in a negative direction, then acceleration is negative. If the speed is decreasing in a negative direction, then acceleration is positive. (You may need to read that a few times slowly to get it.)

Experiment

1. Throw the ball as high as you can and catch it at the same point where you released it. Record approximately how long the ball is in the air. Don't forget to include units.

   Time (t) = _____

2. At what time(s) is the position of the ball 0 ft?

3. At what time(s) is the position of the ball greatest?

4. When is the ball traveling fastest?

5. When is the velocity of the ball 0 ft/s?

6. When is the velocity of the ball positive?

7. When is the velocity of the ball negative?

8. Does the ball have a positive or negative acceleration as it is going up?

9. Does the ball have a positive or negative acceleration as it is going down?

Section 4 – Algebra Practice

There is not enough room to show your work on this worksheet. Please do your work on another sheet of paper. When doing homework this year, include your name, the name of the assignment (typically you will use the section and problem numbers from the book), and the date you started the assignment. Be sure that there are no frayed edges from tearing the sheets out of a spiral binder.
For each problem, make sure that you write the problem down first and then show each step below the previous one. This will help you avoid the evil run-on equation.

Simplify by multiplying by the conjugate pair

1. $\dfrac{1}{\sec\theta+\tan\theta}$

2. $\dfrac{1}{1+\sin x}$

3. $\dfrac{1}{\sqrt{1-\sin x}}$

Write the expression in its simplest form

4. $\dfrac{x^2+3x+2}{x^2+2x}$

5. $\dfrac{(x+1)^2(2x+1)x}{x^4(x+1)(2x+1)^{-2}}$

6. $\dfrac{x^2}{\sqrt{x^5+4x^4}}$

Rewrite trig functions in simplest form

7. $\dfrac{\csc(x)\tan(x)}{\cos(x)}$

8. $\dfrac{\sec(x)\sin(x)}{\csc(x)\cot(x)}$

9. $\dfrac{\cos 2\theta+\sin^2\theta}{\sin^2\theta}$

Use the unit circle to solve for θ (show your work by drawing the unit circle and using special right triangles to figure it out.)

10. $\theta=\sin^{-1}\dfrac{\sqrt{2}}{2}$

11. $\theta=\tan^{-1}1$

12. $\theta=\cos^{-1}\dfrac{\sqrt{3}}{2}$

13. $\sec\theta=\dfrac{2}{\sqrt{3}}$

14. $\cot\theta=\sqrt{3}$

15. $\csc\theta=-2$

Solve for x in terms of y

16. $xy+4=3x-y$

17. $\dfrac{x(y+1)}{y}=1+xy$

18. $y=x^2-2x+1$

Simplify the complex fractions (fractions are complex when either the numerator and/or the denominator contain fractions.

19. $\dfrac{\dfrac{3x+5}{x}}{\dfrac{x}{x+1}}$

20. $\dfrac{\dfrac{1}{x}-x}{\dfrac{(1+x)}{(1-x)}}$

21. $\dfrac{x+\dfrac{1}{2x}-4}{\dfrac{2x}{5}}$

Simplify the expressions so that there are no negative exponents

22. $3x+5x^{-1}+2(y+1)^{-2}$

23. $\dfrac{(x+1)^{-2}(2x+1)^{-4}x}{x^{-4}(x+1)^{2}(2x+1)^{-2}}$

24. $\dfrac{3x+5x^{-1}+y}{\dfrac{1}{x^{2}}+3x^{-2}+4}$

Rewrite expressions so that all terms are in the numerator (Your answers may have negative exponents).

25. $\dfrac{1}{x}+\dfrac{3x}{y^{3}}+\dfrac{9+2x}{x^{-3}y}$

26. $\dfrac{(x+1)}{x^{-9}(x+1)^{3/4}(2x+1)^{-2/5}}$

27. $\dfrac{(x-2y)^{2}}{\sqrt{x^{3}+7x^{4}}}$

Divide using long division or synthetic division

28. $\dfrac{x^{2}-x-12}{x-4}$

29. $\dfrac{n^{3}-2n^{2}+n+2}{n+2}$

Write each expression in terms of ln2 and ln3 in their simplest form

30. $\ln 6$

31. $\log_{3} 6$

32. $\ln\left(\dfrac{128}{81}\right)$

33. $\log_{2} 144$

If $x = u + 1$, make substitutions to rewrite the expressions in terms of u. (Be sure to simplify)

34. $\dfrac{(x-1)^{2}-1}{x}$

35. $\dfrac{10\sqrt{x^{2}-2x+1}}{x^{2}-1}$

Find solutions to the equations using the ZERO function (2nd CALC ZERO) on your calculator. (You do not need to show your work.)

36. $16x^{2}=32x+48$

37. $0=3x^{3}-5x^{2}-8x-1$

Determine if y is positive or negative, given the following value of x without using a calculator.

38. $y=\frac{(x-1)}{(x+1)}; x=.999999999999$

39. $y=\frac{(x-1)x}{\sin(x)}; x=-.0000000000001$

40. $y=\frac{x^{2}}{(x+1)}; x=-.999999999999$

41. $y=\frac{(x^{3}-1)}{(1-x)}; x=1.00000000000001$

42. $y=\frac{(x)\tan(x)}{cos(x)}; x=-0.00000000001$

43. $y=\frac{(x-1)sec(x+1)}{(1-x)csc(x-1)}; x=-1.00000000000001$

Use the helpful websites that you found to write each step to the solution of the derivative and the integral below. These are two concepts that we will learn this year. I don't expect you to understand the solution. I just want to see if you have found a good resource online.

44. $\frac{d}{dx}\left(\frac{1+x}{x^2+3}\right)$

45. $\int_3^4 \left(\frac{3}{5}x^5 + 5x^2\right)(3x^4 + 10x)\,dx$